1. An effect of a certain disease is that a small number of the red blood cells are deformed. Emily has this disease and the deformed blood cells occur randomly at a rate of 2.5 per ml of her blood. Following a course of treatment, a random sample of 2 ml of Emily's blood is found to contain only 1 deformed red blood cell.

Stating your hypotheses clearly and using a 5% level of significance, test whether or not there has been a decrease in the number of deformed red blood cells in Emily's blood.

(Total 6 marks)

2. A web server is visited on weekdays, at a rate of 7 visits per minute. In a random one minute on a Saturday the web server is visited 10 times.

(a) (i) Test, at the 10% level of significance, whether or not there is evidence that the rate of visits is greater on a Saturday than on weekdays. State your hypotheses clearly.

(ii) State the minimum number of visits required to obtain a significant result.

(7)

(b) State an assumption that has been made about the visits to the server.

(1)

In a random two minute period on a Saturday the web server is visited 20 times.

(c) Using a suitable approximation, test at the 10% level of significance, whether or not the rate of visits is greater on a Saturday.

(6)

(Total 14 marks)

3. A test statistic has a Poisson distribution with parameter λ.

Given that

$$H_0 : \lambda = 9, \quad H_1 : \lambda \neq 9$$

(a) find the critical region for the test statistic such that the probability in each tail is as close as possible to 2.5%.

(3)

(b) State the probability of incorrectly rejecting H_0 using this critical region.

(2)
(Total 5 marks)

4. (a) Explain what you understand by

(i) a hypothesis test,

(ii) a critical region.

(3)

During term time, incoming calls to a school are thought to occur at a rate of 0.45 per minute. To test this, the number of calls during a random 20 minute interval, is recorded.

(b) Find the critical region for a two-tailed test of the hypothesis that the number of incoming calls occurs at a rate of 0.45 per 1 minute interval. The probability in each tail should be as close to 2.5% as possible.

(5)

(c) Write down the actual significance level of the above test.

(1)

In the school holidays, 1 call occurs in a 10 minute interval.

(d) Test, at the 5% level of significance, whether or not there is evidence that the rate of incoming calls is less during the school holidays than in term time.

(5)
(Total 14 marks)

5. Bacteria are randomly distributed in a river at a rate of 5 per litre of water. A new factory opens and a scientist claims it is polluting the river with bacteria. He takes a sample of 0.5 litres of water from the river near the factory and finds that it contains 7 bacteria. Stating your hypotheses clearly test, at the 5% level of significance, the claim of the scientist.

(Total 7 marks)

6. Past records from a large supermarket show that 20% of people who buy chocolate bars buy the family size bar. On one particular day a random sample of 30 people was taken from those that had bought chocolate bars and 2 of them were found to have bought a family size bar.

(a) Test at the 5% significance level, whether or not the proportion p, of people who bought a family size bar of chocolate that day had decreased. State your hypotheses clearly.

(6)

The manager of the supermarket thinks that the probability of a person buying a gigantic chocolate bar is only 0.02. To test whether this hypothesis is true the manager decides to take a random sample of 200 people who bought chocolate bars.

(b) Find the critical region that would enable the manager to test whether or not there is evidence that the probability is different from 0.02. The probability of each tail should be as close to 2.5% as possible.

(6)

(c) Write down the significance level of this test.

(1)
(Total 13 marks)

7. Breakdowns occur on a particular machine at random at a mean rate of 1.25 per week.

(a) Find the probability that fewer than 3 breakdowns occurred in a randomly chosen week.

(4)

Over a 4 week period the machine was monitored. During this time there were 11 breakdowns.

(b) Test, at the 5% level of significance, whether or not there is evidence that the rate of breakdowns has changed over this period. State your hypotheses clearly.

(7)
(Total 11 marks)

8. Over a long period of time, accidents happened on a stretch of road at random at a rate of 3 per month.

Find the probability that

(a) in a randomly chosen month, more than 4 accidents occurred,

(3)

(b) in a three-month period, more than 4 accidents occurred.

(2)

At a later date, a speed restriction was introduced on this stretch of road. During a randomly chosen month only one accident occurred.

(c) Test, at the 5% level of significance, whether or not there is evidence to support the claim that this speed restriction reduced the mean number of road accidents occurring per month.

(4)

The speed restriction was kept on this road. Over a two-year period, 55 accidents occurred.

(d) Test, at the 5% level of significance, whether or not there is now evidence that this speed restriction reduced the mean number of road accidents occurring per month.

(7)
(Total 16 marks)

9. (a) Explain what you understand by a critical region of a test statistic.

(2)

The number of breakdowns per day in a large fleet of hire cars has a Poisson distribution with mean $\frac{1}{7}$.

(b) Find the probability that on a particular day there are fewer than 2 breakdowns.

(3)

(c) Find the probability that during a 14-day period there are at most 4 breakdowns.

(3)

The cars are maintained at a garage. The garage introduced a weekly check to try to decrease the number of cars that break down. In a randomly selected 28-day period after the checks are introduced, only 1 hire car broke down.

(d) Test, at the 5% level of significance, whether or not the mean number of breakdowns has decreased. State your hypotheses clearly.

(7)
(Total 15 marks)

10. Vehicles pass a particular point on a road at a rate of 51 vehicles per hour.

(a) Give two reasons to support the use of the Poisson distribution as a suitable model for the number of vehicles passing this point.

(2)

Find the probability that in any randomly selected 10 minute interval

(b) exactly 6 cars pass this point,

(3)

(c) at least 9 cars pass this point.

(2)

After the introduction of a roundabout some distance away from this point it is suggested that the number of vehicles passing it has decreased. During a randomly selected 10 minute interval 4 vehicles pass the point.

(d) Test, at the 5% level of significance, whether or not there is evidence to support the suggestion that the number of vehicles has decreased. State your hypotheses clearly.

(6)
(Total 13 marks)

11. From past records a manufacturer of ceramic plant pots knows that 20% of them will have defects. To monitor the production process, a random sample of 25 pots is checked each day and the number of pots with defects is recorded.

(a) Find the critical regions for a two-tailed test of the hypothesis that the probability that a plant pot has defects is 0.20. The probability of rejection in either tail should be as close as possible to 2.5%.

(5)

(b) Write down the significance level of the above test.

(1)

A garden centre sells these plant pots at a rate of 10 per week. In an attempt to increase sales, the price was reduced over a six-week period. During this period a total of 74 pots was sold.

(c) Using a 5% level of significance, test whether or not there is evidence that the rate of sales per week has increased during this six-week period.

(7)
(Total 13 marks)

12. A single observation x is to be taken from a Poisson distribution with parameter λ. This observation is to be used to test $H_0 : \lambda = 7$ against $H_1 : \lambda \neq 7$.

(a) Using a 5% significance level, find the critical region for this test assuming that the probability of rejecting in either tail is as close as possible to 2.5%.

(5)

(b) Write down the significance level of this test.

(1)

The actual value of x obtained was 5.

(c) State a conclusion that can be drawn based on this value.

(2)
(Total 8 marks)

13. From past records a manufacturer of glass vases knows that 15% of the production have slight defects. To monitor the production, a random sample of 20 vases is checked each day and the number of vases with slight defects is recorded.

(a) Using a 5% significance level, find the critical regions for a two-tailed test of the hypothesis that the probability of a vase with slight defects is 0.15. The probability of rejecting, in either tail, should be as close as possible to 2.5%.

(5)

(b) State the actual significance level of the test described in part (*a*).

(1)

A shop sells these vases at a rate of 2.5 per week. In the 4 weeks of December the shop sold 15 vases.

(c) Stating your hypotheses clearly test, at the 5% level of significance, whether or not there is evidence that the rate of sales per week had increased in December.

(6)
(Total 12 marks)

1.	H_0:λ = 2.5 (or λ = 5) H1: λ < 2.5 (or λ < 5)	λ or μ	B1B1
	X ~ Po(5)		M1
	P($X \le 1$) = 0.0404 or CR $X \le 1$		A1
	[0.0404<0.05] this is significant or reject H_0 or it is in the critical region		M1
	There is evidence of a <u>decrease</u> in the (mean) <u>number/rate</u> of <u>deformed blood cells</u>		A1

<u>Note</u>

1st B1 for H_0 must use lambda or mu; 5 or 2.5.

2nd B1 for H_1 must use lambda or mu; 5 or 2.5

1st M1 for use of Po(5) may be implied by probability(must be used not just seen)

eg. P (X = 1) = 0.0404 – … would score M1 A0

1st A1 for 0.0404 seen or correct CR

2nd M1 for a correct statement (this may be contextual) comparing their probability and 0.05 (or comparing 1 with their critical region).
Do not allow conflicting statements.

2nd A1 is not a follow through. Need the word decrease, number or rate and deformed blood cells for contextual mark.

If they have used $\neq$ in H1 they could get B1 B0 M1 A1 M1A0 mark as above except they gain the

1stA1 for P($X \le 1$) = 0.0404 or CR $X \le 0$

2nd M1 for a correct statement (this may be contextual) comparing their probability and 0.025 (or comparing 1 with their critical region)

They may compare with 0.95 (one tail method) or 0.975 (one tail method) Probability is 0.9596.

[6]

2.	(a)	(i)	$H_0 : \lambda = 7$ $H_1 : \lambda > 7$	B1	
			X = number of visits. $X \sim \text{Po}(7)$	B1	
			$P(X \geq 10) = 1 - P(X \leq 9) = 0.1695$ $1 - P(X \leq 10) = 0.0985$ $1 - P(X \leq 9) = 0.1695$	M1	
			CR $X \geq 11$	A1	
			$0.1695 > 0.10$, CR $X \geq 11$ Not significant or it is not in the critical region or do not reject H_0	M1	
			The rate of visits on a Saturday is not greater/ is unchanged	A1 no ft	
		(ii)	$X = 11$	B1	7
	(b)		(The visits occur) randomly/ independently or singly or constant rate	B1	7
	(c)		$[H_0 : \lambda = 7$ $H_1 : \lambda > 7$ (or $H_0 : \lambda = 14$ $H_1 : \lambda > 14)]$		
			$X \sim$N;(14,14)	B1;B1	
			$P(X \geq 20) = P\left(z \geq \frac{19.5 - 14}{\sqrt{14}}\right)$ +/– 0.5, stand	M1 M1	
			$= P(z \geq 1.47)$ $= 0.0708$ or $z = 1.2816$	A1dep both M	
			$0.0708 < 0.10$ therefore significant. The rate of visits is greater on a Saturday	A1dep 2nd M	6

[14]

3.	(a)	$X \sim \text{Po}(9)$ may be implied by calculations in part a or b	M1	
		$P(X \leq 3) = 0.0212$ $P(X \geq 16) = 0.0220$		
		CR $X \leq 3$; $\cup X \geq 16$	A1;A1	3

M1 for using Po (9) – other values you might see which imply Po (9) are 0.0550, 0.0415, 0.9780, 0.9585, 0.9889, 0.0111,0.0062 or may be assumed by at least one correct region.
A1 for $X \leq 3$ or $X < 4$ condone c1 or CR instead of X
A1 for $X \geq 16$ or $X > 15$

They must identify the critical regions at the end and not just have them as part of their working. Do not accept $P(X \leq 3)$ etc gets A0

(b) P(rejecting Ho) = 0.0212 + 0.0220 M1
= 0.0432 or 0.0433 A1cao 2

if they use 0.0212 and 0.0220 they can gain these marks regardless of the critical regions in part a. If they have not got the correct numbers they must be adding the values for their critical regions. (both smaller than 0.05)
You may need to look these up. The most common table values for lambda = 9 are in this table

x	2	3	4	5	14	15	16	17	18
	0.0062	0.0212	0.0550	0.1157	0.9585	0.9780	0.9889	0.9947	0.9976

A1 awrt 0.0432 or 0.0433

Special case
If you see 0.0432 / 0.0433 and then they go and do something else with it eg 1 – 0.0432 award M1 A0

[5]

4. (a) (i) A hypothesis test is a mathematical procedure to <u>examine a value of a population parameter</u> proposed by <u>the null hypothesis compared</u> with an alternative hypothesis. B1

B1 Method for deciding between 2 hypothesis.

(ii) The critical region is the range of values **or** a test statistic or region where the test is significant B1g
that would lead <u>to the rejection of H_0</u>. B1h 3

B1 range of values. This may be implied by other words. Not region on its own

B1 which lead you to <u>reject H_0</u>

Give the first B1 if only one mark awarded.

(b)	Let X represent the number of incoming calls : X ~ Po(9)	B1	
	From table $P(X \geq 16) = 0.0220$	M1 A1	
	$P(x < 3) = 0.0212$	A1	
	Critical region ($x \leq 3$ or $x \geq 16$)	B1	5

B1 using $P_o(9)$

M1 attempting to find $P(X \geq 16)$ or $P(X \leq 3)$

A1 0.0220 or $P(X \geq 16)$

A1 0.0212 or $P(X \leq 3)$
These 3 marks may be gained by seeing the numbers in part c

B1 correct critical region

A completely correct critical region will get all 5 marks.
Half of the correct critical region eg $x \leq 3$ or $x \geq 17$ say would get B1 M1 A0 A1 B0 if the M1 A1 A1 not already awarded.

(c)	Significance level = 0.0220 + 0.0212 = 0.0432 or 4.32%	B1	1

B1 cao awrt 0.0432

(d)	$H_0 : \lambda = 0.45$; $H_1 : \lambda < 0.45$ (accept : $H_0 : \lambda = 4.5$; $H_1 : \lambda < 4.5$)	B1	
	Using X ~ Po(4.5)	M1	
	$P(X \leq 1) = 0.0611$ CR $X \leq 0$ awrt 0.0611	A1	
	$0.0611 > 0.05$. $1 \geq 0$ or 1not in the critical region	M1	
	There is evidence to Accept H_0 or it is not significant	B1cao	5
	There is no evidence that there are less calls during school holidays.		

B1 may use λ or μ. Needs both H_0 and H_1

M1 using $P_o(4.5)$

A1 correct probability or CR only

M1 correct statement based on their probability , H_1 and 0.05
or a correct contextualised statement that implies that.

B1 this is not a follow through .Conclusion in context.
Must see the word **calls** in conclusion

If they get the correct CR with no evidence of using $P_o(4.5)$
they will get M0 A0

SC If they get the critical region $X \leq 1$ they score M1 for rejecting H_0 and B1 for concluding the rate of calls in the holiday is lower.

[14]

5. One tail test

Method 1

$H_0: \lambda = 5$ ($\lambda = 2.5$)		may use λ or μ	B1	
$H_1: \lambda > 5$ ($\lambda > 2.5$)			B1	
$X \sim \text{Po}(2.5)$		may be implied	M1	
$P(X \geq 7) = 1 - P(X \leq 6)$ $= 1 - 0.9858$	$[P(X \geq 5) = 1 - 0.8912 = 0.1088]$ $P(X \geq 6) = 1 - 0.9580 = 0.0420$	att $P(X \geq 7)$ | $P(X \geq 6)$	M1 A1	
$= 0.0142$	CR $X \geq 6$	awrt 0.0142		
$0.0142 < 0.05$	$7 \geq 6$ or 7 is in critical region or 7 is significant		M1	

(Reject H_0.) There is significant evidence at the 5% significance level that the factory is polluting the river with bacteria. B1 7

or

The scientists claim is justified

Method 2

$H_0: \lambda = 5$ ($\lambda = 2.5$)		may use λ or	B1	
$H_1: \lambda > 5$ ($\lambda > 2.5$)			B1	
$X \sim \text{Po}(2.5)$		may be implied	M1	
$P(X < 7)$	$[P(X < 5) = 0.8912]$ $P(X < 6) = 0.9580$	att $P(X < 7)$ | $P(X < 6)$	M1A1	
$= 0.9858$	CR $X \geq 6$	wrt 0.986		
$0.9858 > 0.95$	$7 \geq 6$ or 7 is in critical region or 7 is significant		M1	

(Reject H_0.) There is significant evidence at the 5% significance level that the factory is polluting the river with bacteria. B1

or

The scientists claim is justified

Two tail test

Method 1

H_0: $\lambda = 5$ ($\lambda = 2.5$) may use λ or B1
H_1: $\lambda \neq 5$ ($\lambda \neq 2.5$) B0

$X \sim \text{Po}(2.5)$ M1

$P(X \geq 7) = 1 - P(X \leq 6)$ $= 1 - 0.9858$	$[P(X \geq 6) = 1 - 0.9580 = 0.0420]$ $P(X \geq 7) = 1 - 0.9858 = 0.0142$	att $P(X \geq 7)$	$P(X \geq 7)$	M1
$= 0.0142$	CR $X \geq 7$	awrt 0.0142		A1
$0.0142 < 0.025$	$7 \geq 7$ or 7 is in critical region or 7 is significant			M1

(Reject H_0.) There is significant evidence at the 5% significance level that the factory is polluting the river with bacteria. B1
or
The scientists claim is justified

Method 2

H_0: $\lambda = 5$ ($\lambda = 2.5$) may use λ or μ B1
H_1: $\lambda \neq 5$ ($\lambda \neq 2.5$) B0

$X \sim \text{Po}(2.5)$ M1

$P(X < 7)$	$[P(X < 6) = 0.9580]$ $P(X < 7) = 0.9858$	att $P(X < 7)$	$P(X < 7)$	M1A1
$= 0.9858$	CR $X \geq 7$	awrt 0.986		
$0.9858 > 0.975$	$7 \geq 7$ or 7 is in critical region or 7 is significant			M1

(Reject H_0.) There is significant evidence at the 5% significance level that the factory is polluting the river with bacteria. B1
or
The scientists claim is justified

[7]

6. (a) $H_0 : p = 0.20$, $H_1 : p < 0.20$ B1, B1
Let X represent the number of people buying family size bar.
$X \sim B(30, 0.20)$
$P(X \leq 2) = 0.0442$ or $P(X \leq 2) = 0.0442$ awrt 0.044 M1A1
$P(X \leq 3) = 0.1227$
CR $X \leq 2$
$0.0442 < 5\%$, so significant. Significant M1
There is evidence that the no. of family size bars sold is lower than usual. A1 6

(b)	$H_0 : p = 0.02$, $H_1: p \neq 0.02$	$\lambda = 4$ etc ok both	B1		
	Let Y represent the number of gigantic bars sold.		B1		
	$Y \sim B(200, 0.02) \Rightarrow Y \sim Po(4)$	can be implied below	M1		
	$P(Y = 0) =$ **0.0183** and $P(Y \leq 8) =$ **0.9786** $\Rightarrow P(Y \geq 9) =$ **0.0214**	first, either	B1,B1		
	Critical region $Y = 0 \cup Y \geq 9$	$Y \leq 0$ ok	B1,B1	6	
	N.B. Accept exact Bin: 0.0176 and 0.0202				
(c)	Significance level = 0.0183 + 0.0214 = 0.0397	awrt 0.04	B1	1	
					[13]

7. (a) Let X represent the number of breakdowns in a week.

$X \sim P_o(1.25)$ *Implied*		B1
$P(X < 3) = P(0) + P(1) + P(2)$ or $P(X \leq 2)$		M1
$= e^{-1.25}\left(1 + 1.25 + \frac{(1.25)^2}{2!}\right)$		A1
$= 0.868467$ *awrt 0.868 or 0.8685*		A1	4

(b)

H_0: $\lambda = 1.25$; H_1: $\lambda \neq 1.25$ (or $H_o : \lambda = 5$; $H_1 : \lambda \neq 5$)	λ or μ	B1 B1	
Let Y represent the number of breakdowns in 4 weeks			
Under H_0, $Y \sim P_0(5)$ *may be implied*		B1	
$P(Y \geq 11) = 1 - P(Y \leq 10)$ or $P(X \geq 11) = 0.0137$ *One needed for M*		M1	
$P(X \geq 10) = 0.0318$			
$= 0.0137$ CR $X \geq 11$		A1	
$0.0137 < 0.025$, $0.0274 < 0.05$, $0.9863 > 0.975$, $0.9726 > 0.95$ or $11 \geq 11$ *any .allow %* *ft from H_1*		M1	
Evidence that the rate of breakdowns has changed / decreased *Context* *From their p*		B1ft	7

[11]

8.	Let X represent number of accidents/month $\therefore X \sim P_0(3)$		B1	
(a)	$P(X > 4) = 1 - P(X \leq 4)$; $= 1 - 0.8513 = \underline{0.1847}$		M1; A1	3
(b)	Let Y represent number of accidents in 3 months $\therefore Y \sim P_0(3 \times 3 = 9)$		B1	
		Can be implied		
	$P(Y > 4) = 1 - 0.0550 = \underline{0.9450}$		B1	2
(c)	$H_0: \lambda = 3$; $H_1: \lambda < 3$		B1	
		both		
	$\alpha = 0.05$ $P(X \leq 1/\lambda = 3) = 0.1991$; > 0.05		B1 M1	
		detailed; allow B0B1M1 (0.025) A0		
	$\therefore$ Insufficient evidence to support the claim that the mean number of accidents has been reduced.		A1ft	4
	(NB: CR: $X \leq 0$; $X = 1$ not in CR; same conclusion $\Rightarrow$ B1, M1, A1)			
(d)	$H_0: \lambda\ 24 \times 3 = 72$; $H_1: \lambda < 72$		B1	
		can be implied $\lambda = 72$		
	$\alpha = 0.05 \Rightarrow$ CR: $\delta < -1.6449$		B1 B1	
		both H_0 & H_1 *-1.6449*		
	Using Normal approximation with $\mu = \sigma^2 = 72$		B1	
		Can be implied		
	$\delta = \dfrac{55.5 - 72}{\sqrt{72}} = -1.94454\ldots$		M1 A1	
		Stand. with ± 0.5, $\mu = \sigma$ *AWRT $-1.94/5$*		
	Since $-1.944\ldots$ is in the CR, H_0 is rejected. There is evidence that the restriction has reduced the number of accidents		A1ft	7
		Context & clear evidence		
	Aliter (d)			
	$p = 0.0262 < 0.05$			
		AWRT 0.026 equn to -1.6449		

[16]

9.	(a)	A range of values of a test statistic such that if a value of the test statistic obtained from a particular sample lies in the critical region, then the null hypothesis is rejected (or equivalent).		B1B1	2
	(b)	P(X < 2) = P(X = 0) + P(X = 1)	both	M1	
		$= e^{-\frac{1}{7}} + \dfrac{e^{-\frac{1}{7}}}{7}$	both	A1	
		= 0.990717599… = 0.9907 to 4 sf *awrt 0.991*		A1	3
		$X \sim$ P$(14 \times \frac{1}{7})$ = P(2)		B1	
		P($X \le 4$) = 0.9473 *Correct inequality, 0.9473*		M1A1	3
		H_0: $\lambda = 4$, H_1 : $\lambda < 4$ *Accept μ & H_0: $\lambda = \frac{1}{7}$, H_1: $\lambda < \frac{1}{7}$*		B1B1	
		$X \sim$ P(4) *Implied*		B1	
		P($X \le 1$) = 0.0916 > 0.05, *Inequality 0.0916*		M1A1	
		So insufficient evidence to reject null hypothesis		A1	
		Number of breakdowns has not significantly decreased		A1	7

[15]

10.	(a)	Vehicles pass at random / one at a time / independently / at a constant rate Any 2&context		B1B1dep	2
	(b)	X is the number of vehicles passing in a 10 minute interval,			
		$X \sim \text{Po}\left(\dfrac{51}{60} \times 10\right)$ = Po(8.5) *Implied Po(8.5)*		B1	
		P(X = 6) = $\dfrac{8.5^6 e^{-8.5}}{6!}$, = 0.1066 (or 0.2562 – 0.1496 = 0.1066) *Clear attempt using 6, 4dp*		M1A1	3
	(c)	P($X \ge 9$) = 1 – P($X \le 8$) = 0.4769 *Require 1 minus and correct inequality*		M1A1	2

	(d)	H_0: $\lambda = 8.5$, H_1: $\lambda < 8.5$	B1ft,B1ft	
		One tailed test only for alt hyp		
		$P(X \leq 4 \mid \lambda = 8.5) = 0.0744$, > 0.05	M1A1	
		$X \leq 4$ for method, 0.0744		
		(**Or** $P(X \leq 3 \mid \lambda = 8.5) = 0.0301$, < 0.05 so CR $X \leq 3$ correct CR	M1**,**A1)	
		Insufficient evidence to reject H_0,	'Accept' M1	
		so no evidence to suggest number of vehicles has decreased.	Context A1ft	6
				[13]
11.	(a)	Let X represent the number of plant pots with defects, $X \sim B(25,0.20)$	B1	
		Implied		
		$P(X \leq 1) = 0.0274$, $P(X \geq 10) = 0.0173$	M1A1A1	
		*Clear attempt at both tails required**,** 4dp*		
		Critical region is $X \leq 1$, $X \geq 10$	A1	5
	(b)	Significance level = 0.0274 + 0.0173 = 0.0447	B1 cao	1
		Accept % 4dp		
	(c)	H_0: $\lambda = 10$, H_1: $\lambda > 10$ (or H_0: $\lambda = 60$, H_1: $\lambda > 60$)	B1B1	
		Let Y represent the number sold in 6 weeks, under H_0, $Y \sim Po(60)$		
		$P(Y \geq 74) \approx P(W > 73.5)$ where $W \sim N(60,60)$	M1A1	
		± 0.5 for cc, 73.5		
		$\approx P\left(Z \geq \frac{73.5 - 60}{\sqrt{60}}\right) = P(Z > 1.74) =$, 0.047 – 0.0409 <0.05	M1,A1	
		Standardise using 60 $\sqrt{60}$		
		Evidence that rate of sales per week has increased.	A1ft	7
				[13]
12.	(a)	$X \sim Po(7)$	B1	
		$P(X \leq 2) = 0.0296$	B1	
		$P(X \geq 13) = 1 - 0.9370 = 0.0270$	M1 A1	
		Critical region is $(X \leq 2) \cup (X \geq 13)$	A1	5
	(b)	Significance level = 0.0296 + 0.0270 = 0.0566	B1	1
	(c)	$x = 5$ is not the critical region $\Rightarrow$ insufficient evidence to reject H_0	M1 A1	2
				[8]

13.	(a)	X = no. of vases with defects $\quad X \sim B(20, 0.15)$	B1	
		$P(X \leq 0) = 0.0388$		
		Use of tables to find each tail	M1	
		$P(X \leq 6) = 0.9781 \quad \therefore P(X \geq 7) = 0.0219$	M1	
		$\therefore$ critical region is $X \leq 0$, or $X \geq 7$	A1 A1	5
		Significance level = $P(X \leq 0) + P(X \geq 7) = 0.0388 + 0.0219 = 0.0607$	(B1)	1
		H_0: $\lambda = 2.5$, $\quad H_1$: $\lambda > 2.5 \quad$ [or H_0: $\lambda = 10$, $\quad H_1$: $\lambda > 10$]	B1, B1	
		Y = no. sold in 4 weeks. $\quad$ Under H_0 $Y \sim Po(10)$	M1	
		$P(Y \geq 15) = 1 - P(Y \leq 14) =, 1 - 0.9165 = 0.0835$	M1, A1	
		More than 5% so not significant. Insufficient evidence of an increase in the rate of sales.	A1	6

[12]

1. Candidates seemed better prepared for this type of question than in previous years. Marks were often lost for not using λ or μ in the hypotheses and for not putting the conclusion into context. A significant minority of candidates found P(X = 1) instead of P(X≤1) but only a few candidates chose the critical region route.

2. In Part (a) there are a sizeable number of candidates who are not using the correct symbols in defining their hypotheses although the majority of candidates recognised Po(7).

For candidates who attempted a critical region there were still a number who struggled to define it correctly for a number of reasons:

- Looking at the wrong tail and concluding X≤3.
- Incorrect use of > sign when concluding 11 - not appreciating that this means ≥12 for a discrete variable.
- Not knowing how to use probabilities to define the region correctly and concluding 10 or 12 instead of 11.

The candidates who opted to calculate the probability were generally more successful.

A minority still try to calculate a probability to compare with 0.9. This proved to be the most difficult route with the majority of students unable to calculate the probability or critical region correctly. We must once again advise that this is not the recommended way to do this question. There are still a significant number who failed to give an answer in context although fewer than in previous sessions.

Giving the minimum number of visits needed to obtain a significant result proved challenging to some and it was noticeable that many did not use their working from part (a) or see the connection between the answer for (i) and (ii) and there were also number of candidates who did not recognise inconsistencies in their answers.

A number of candidates simply missed answering part (b) but those who did usually scored well.

There were many excellent responses in part (c) with a high proportion of candidates showing competence in using a Normal approximation, finding the mean and variance and realising that a continuity correction was needed. Marks were lost, however, for not including 20, and for not writing the conclusion in context in terms of the **rate** of visits being **greater**. Some candidates attempted to find a critical value for X using methods from S3 but failing to use 1.2816. There were a number of candidates who calculated P(X = 20) in error.

3. Whilst many candidates knew what they were doing in part (a) they lost marks because they left their answers as P(X ≤ 3) etc and did not define the critical regions. A few candidates were able to get the figures 0.0212 and 0.0220 but then did not really understand what this meant in terms of the critical value. A critical region of $X \geq 15$ was common.

Part (b) was poorly answered. The wording "incorrectly rejecting H_0" confused many candidates. They often managed to get to 0.432 but then they took this away from 0.5 or occasionally 1. It was not uncommon for this to be followed by a long paragraph trying to describe what they had done.

4. This question appeared to be difficult for many candidates with a large proportion achieving less than half the available marks.

(a) The majority of candidates were unable to give an accurate description of a hypothesis test as a method of deciding between 2 hypotheses. There were more successful definitions of a critical region but many candidates achieved only 0 or 1 of the 3 available marks. Common errors included too much re-use of the word region without any expansion on it. Even those who could complete the rest of the question with a great deal -of success could not describe accurately what they were actually doing.

(b) Although most of those attempting this part of the question realised that a Poisson distribution was appropriate there was a sizeable number who used a Binomial distribution. Again, the most common problem was in expressing and interpreting inequalities in order to identify the critical regions. Many found the correct significance level but struggled to express the critical region correctly. Answers with 15 were common and some candidates even decided that 4 to 15 was the CR.

(c) Those candidates that identified the correct critical regions were almost always able to state the significance level correctly, as were some who had made errors in stating these regions. Some still gave 5% even with part (b) correct.

(d) Candidates who had used a Binomial distribution in part (b), and many of those who had not, used p instead of λ in stating the hypotheses and went on to obtain a Binomial probability in this part of the question. In obtaining $P(X \leq 1)$ some used Po(9) from part (b) instead of Po(4.5). Most of those achieving the correct statement (failing to reject H_0) were able to place this in a suitable contextualised statement. There were some candidates who still tried to find $P(X = 1)$ rather than $P(X < 1)$.

5. The majority of candidates found this question straightforward. They were most successful if they used the probability method and compared it with 0.05. Those who attempted to use 95% were less successful and this is not a recommended route for these tests.
Most candidates knew how to specify the hypotheses with most candidates using 2.5 rather than 5. Some candidates used p, or did not use a letter at all, in stating their hypotheses, but most of the time they used λ. A minority found $P(X = 7)$ and some worked with Po(5).
If using the critical region method, not all candidates showed clearly, either their working, or a comparison with the value of 7 and the CR $X \geq 7$.
A sizeable minority of candidates failed to put their conclusion back into the given context. Reject H_0 is not sufficient.

6. Weaker candidates found this question difficult and even some otherwise very strong candidates failed to attain full marks. Differentiating between hypothesis testing and finding critical regions and the statements required, working with inequalities and placing answers in context all caused problems. In part (a) a large number of candidates were able to state the hypotheses correctly but a sizeable minority made errors such as missing the p or using an alternative (incorrect) symbol. Some found $P(X = 2)$ instead of $P(X \leq 2)$ and not all were able to place their solution in the correct context. Not all candidates stated the hypotheses they were using to calculate the critical regions in part (b). In a practical situation this makes these regions pointless. The lower critical region was identified correctly by many candidates but many either failed to realise that $P(X \leq 8)=0.9786$ would give them the correct critical region and/or that this is $X \geq 9$. The final part was often correct.

7. Most candidates answered Part (a) correctly. A small number of candidates calculated the probability for less than or equal to 3 although a minority thought that dividing by 0! in P(X = 0) gave zero. In part (b) carrying out the hypothesis test was more challenging though there was clear evidence that candidates had been prepared for this type of question. However, using p instead of λ or μ, when stating the hypotheses, was often seen and incorrectly stating H1 as $\lambda >$ 1.25 or 5 also lost marks. Many candidates calculated P(X $\leq$ 11) instead of looking at P(X $\geq$ 11). A diagram would have helped them or the use of the phrase "a result as or more extreme than that obtained". Those who used the critical region approach made more errors. Some candidates correctly calculated the probability and compared it with 0.025 but were then unsure of the implications for the hypotheses. A few candidates used a 2-tailed hypothesis but then used 0.05 rather than 0.025 in their comparison. Most candidates gave their conclusions in context.

8. For those candidates that could interpret 'more than 4 accidents occurred' correctly parts (a) and (b) were a good source of marks. Part (b) was often well answered and many candidates gained full marks. In part (c) incorrect hypotheses and ignoring the continuity correction were the common errors coupled with poor use of the appropriate significance test. Candidates need to have a simple algorithm at their fingertips to deal with tests of significance.

9. In part (a) many candidates struggled to explain the concept of a critical region, although some gave a correct definition as the range of values where the null hypothesis is rejected. Many correct solutions were seen for parts (b) and (c). However the weaker candidates were not able to translate the concept 'at most 4 breakdowns' to the correct inequality. In part (d), as with Q3, many candidates successfully performed the required hypothesis test using a probability method. Again, there was a sizeable number of candidates who incorrectly found P(X=1) and compared this probability with the significance level. Again, a minority of candidates decided to approach this question using the critical region strategy. Marks were lost if candidates did not give evidence of their chosen critical region.

10. Candidates were able to express two conditions for a Poisson distribution in context with vehicles passing by a particular point on the road. Many candidates then answered part (b) and (c) correctly. In part (d) a majority of candidates was able to give a full solution by either using a probability or critical region approach to their hypothesis test.

11. Many candidates found this question difficult. A few candidates failed to look for the two tails in part (a) and, of those that did, many chose any value that was less than 2.5% rather than the closest value. Many identified the correct probability for the upper region, but then failed to interpret this as a correct critical region. Marks were lost by those who failed to show which values they had extracted from the tables to obtain their results. Nearly all of those who achieved full marks in part (a) answered part (b) correctly.

In part (c) weaker candidates had difficulty in stating hypotheses correctly and then attempted to use a Poisson distribution with a parameter obtained from dividing 74 by 6. However, the best candidates realised that a normal approximation was appropriate, with the most common error being an incorrect application of the continuity correction. Most solutions were placed in context.

12. Many good candidates lost marks carelessly by failing to show detailed working, even if they arrived at the correct critical region, and the statement of the region was often missing or stated as a probability. Some centres had candidates who were trying to follow the book method too closely and did not demonstrate a clear understanding of the concept of significance.

13. No Report available for this question.